

Bivariate Weibull Distributions Applied to Maintenance Modeling

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Abstract

The main idea of the paper is to determine a bivariate Weibull distribution for maintenance data, with Weibull marginal repartitions, generated from a bivariate standard normal repartition, each component (mileage and costs) having a normal distribution. The marginal univariate Weibull distributions were obtained from the values of standard cumulative distribution functions of the normal variables, with a correlation coefficient ρ . The values of the cumulative distribution function, derived from normal, follow an uniform continuous distribution on $[0; 1]$. The paper proposes procedures of calculus for bivariate Weibull distributions to enhance the modeling of the maintainability vs. the normal distribution. The calculus of the multivariate Weibull cumulative distribution function (CDF) is based on standard bivariate normal distribution, with a change of variables, which maintains and transfers the features of the former. The CDF of a univariate Weibull distribution was obtained using erf function, which can convert to a new variable, uniform distributed. Each univariate normal distribution with two parameters is transformed in a Weibull one with scale and shape parameters are estimated the four parameters of the joint Weibull distribution. The estimate of the correlation coefficient, ρ , of the initial normal marginal distributions, was considered as a measure of the dependence to determine the bivariate probability density function (PDF). The maximum likelihood method was applied to the bivariate Weibull probability density function to obtain an estimate of dependence parameter, ρ , necessary for the calculus of joint Weibull CDF.

Keywords: bivariate Weibull distribution, maintenance-costs

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